v is the speed of sound, m/sec;
M is the molecular weight, g;
P is the pressure;
V is the specific volume;
C_P/C_V is the ratio of the specific heats;
σ is the coefficient of surface tension.

Subscripts

c is critical;
l is liquid;
v is vapor;
ef is effective.

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INVESTIGATION OF TRANSITION BOILING UNDER NONSTATIONARY COOLING CONDITIONS

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The method of nonstationary cooling is extensively used at present to investigate boiling. This method was applied successfully in a study of film boiling of cryogenic liquids [1-3]. Processing the test data in this case consists of determining the heat flux q and the heat-exchange surface temperature T_W according to the known dependence of the temperature at the site of thermocouple installation T_0 on the time τ. The values of q and T_W were determined either from the heat balance equation [3], or by the method of a regular regime [1], or from the solution of the inverse problem of heat conduction [2]. All the methods mentioned yield the same formula to compute the heat flux in an investigation of film boiling on working sections for which the condition $Bi < 0.1$ is satisfied:

$$q = -\rho c\delta \frac{dT_0}{d\tau}. \tag{1}$$

The method of nonstationary cooling is used in a number of papers [3, 4, 5] to investigate transition boiling. However, in this case a computation by means of (1) can result in substantial methodological errors because of the rapid change in the coefficient of heat transfer and the nonuniformity of the temperature field. It is

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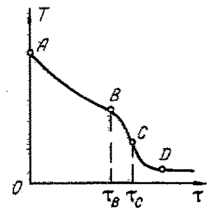

Fig. 1. Typical temperature curve of body cooling in a boiling liquid.

impossible to discard arbitrarily the higher terms in the expansions in the solution of the inverse problem of heat conduction [6, 7]

$$T_0 - T_W = -\sum_{n=1}^{\infty} E_n \frac{d^n T_0}{d\tau^n}, \tag{2}$$

$$q = -\sum_{n=1}^{\infty} F_n \frac{d^n T_0}{d\tau^n} \tag{3}$$

in an investigation of transition boiling since they can be commensurate with the first term in the series. Meanwhile, multiple numerical differentiation of the experimental curve $T_0(\tau)$ results in large errors. Hence, the direct application of (2) and (3) for processing the test data in transition boiling is impossible.

The problem in this paper is to obtain computational formulas to determine q and T_W in an investigation of heat transfer during transition boiling by the method of nonstationary cooling. To this end, it is proposed to approximate the whole cooling temperature curve $T_0(\tau)$ by a single smooth function, to obtain a general expression for the derivative $d^n T_0/d\tau^n$, and to convert (2) and (3) into infinite number series of the approximation parameters.

A typical time dependence of the temperature during nonstationary cooling of a body in a boiling liquid is shown in Fig. 1. The section AB corresponds to film boiling, BC to transition, and CD to nucleate boiling. A sequential interchange of film and nucleate boiling is observed at each point of the heat exchange surface during transition boiling. Hence, the dependence of the heat flux on the temperature head can be represented as a superposition of the appropriate dependences for nucleate and film boiling taken with the weight coefficients f and (1 − f) [8]. The value of f varies from 0 to 1 with the course of time in the nonstationary cooling of a body in a boiling liquid. Starting from this, a function should be selected as the single approximate dependence for the curve $T_0(\tau)$, which would satisfy the equation [9]

$$q = \alpha_{fl} (T_W - T_s) + q_l, \tag{4}$$

on the film boiling section AB, the equation [10]

$$q = A (T_W - T_s)^n \tag{5}$$

on the nucleate boiling section CD, and would make the transition from (4) to (5) smoothly on the section BC.

For a thin-walled body when $T_0 \approx T_W$, we obtain the dependence $T_W(\tau)$ on the film boiling section by integrating (1) with (4) or (5) taken into account

$$T_{fl}(\tau) = T_s - \frac{q_l}{\alpha_{fl}} + \left(\Delta T_1 + \frac{q_l}{\alpha_{fl}}\right) e^{-\frac{\alpha_{fl} \cdot \tau}{\rho c \delta}} \tag{6}$$

and on the nucleate boiling section

$$T_{nu}(\tau) = T_s + \left[\frac{(n-1)A}{\rho c \delta}(\tau - \tau_1) + \frac{1}{\Delta T_1^{n-1}}\right]^{-\frac{1}{n-1}}, \tag{7}$$

where ΔT_1 is the temperature heat at some time τ_1 on the nucleate boiling section.

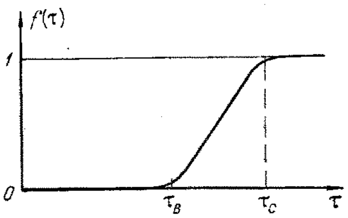

Fig. 2. Transition function f(τ).

However, it is impossible to use the function (7) in a single approximate dependence since it has physical meaning only in the domain

$$\tau > \tau_2 = \tau_1 - \frac{\rho c\delta}{(n-1)A\Delta T_1^{n-1}}, \tag{8}$$

and becomes infinite for $\tau = \tau_2$. For the real dependence $T_0(\tau)$ the values $\tau \leq \tau_2$ are possible even within the limits of the section BC. Hence, instead of the dependence (5) in the nucleate boiling domain CD, a linear approximation of the section of the boiling curve abutting the transition domain must be used:

$$q = \alpha_{\mathrm{nu}}(T_{\mathrm{W}} - T_{\mathrm{s}} - \Delta T_0), \tag{9}$$

which yields after substitution into (1) and subsequent integration

$$T_{\mathrm{nu}}(\tau) = T_{\mathrm{s}} + \Delta T_0 + (\Delta T_1 - \Delta T_0)\exp\left[-\frac{\alpha_{\mathrm{nu}}(\tau - \tau_1)}{\rho c\delta}\right]. \tag{10}$$

If a certain function f(τ) which increases smoothly from 0 to 1 is selected, then the dependence

$$T_0(\tau) = (1 - f)T_{\mathrm{fl}}(\tau) + fT_{\mathrm{nu}}(\tau) \tag{11}$$

can have a section with a positive derivative in the general case, while the experimental dependence $T_0(\tau)$ decreases monotonically everywhere. Hence, the construction of an approximating function for the derivative $dT_0/d\tau$ has a better foundation physically. From (6) we have

$$\left(\frac{dT_{\mathrm{W}}}{d\tau}\right)_{\mathrm{fl}} = -\frac{\alpha_{\mathrm{fl}}\Delta T_1 + q_l}{\rho c\delta}\exp\left[-\frac{\alpha_{\mathrm{fl}}\tau}{\rho c\delta}\right] \tag{12}$$

on the film boiling section, and from (10)

$$\left(\frac{dT_{\mathrm{W}}}{d\tau}\right)_{\mathrm{nu}} = -\frac{\alpha_{\mathrm{nu}}(\Delta T_1 - \Delta T_0)}{\rho c\delta}\exp\left[-\frac{\alpha_{\mathrm{nu}}(\tau - \tau_1)}{\rho c\delta}\right] \tag{13}$$

on the nucleate boiling section.

We select the approximating dependence for $dT_0/d\tau$ in such a way that it would have a structural form analogous to (12) on the section AB:

$$\tau \to 0;\quad \frac{dT_0}{d\tau} \to -D_1 e^{-B_1\tau}, \tag{14}$$

and a form analogous to (13) on the section CD:

$$\tau \to \infty;\quad \frac{dT_0}{d\tau} \to D_2 e^{-B_2\tau}. \tag{15}$$

The smooth transition from (14) to (15) on the section BC is assured by using some function f(τ) whose form is shown in Fig. 2. This, smooth, infinitely differentiable function tends asymptotically to 0 for $\tau < \tau_B$ and to 1 for $\tau > \tau_C$. The following functions, for instance:

$$f_1(\tau) = \frac{1}{2} + \frac{1}{2}\operatorname{erf}\{B(\tau - \tau_0)\}; \tag{16}$$

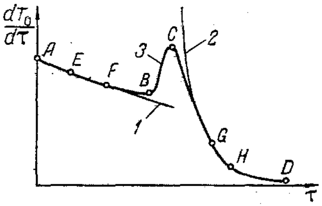

Fig. 3. Approximation curve for $dT_0/d\tau$: 1) from (14); 2) from (15), and 3) from (20).

$$f_2(\tau) = \frac{1}{2} + \frac{1}{2}\,\mathrm{th}\left\{\frac{B}{2}(\tau - \tau_0)\right\} = \frac{1}{1 + e^{-B(\tau-\tau_0)}}; \tag{17}$$

$$f_3(\tau) = \frac{1}{2} + \frac{1}{\pi}\,\mathrm{Arctg}\,\{B(\tau - \tau_0)\}, \tag{18}$$

where $\tau_0 \in (\tau_B, \tau_C)$, can be selected as the dependence $f(\tau)$.

The dependence (17) is taken for the function $f(\tau)$ in this paper, and the weight functions have the form

$$f = \frac{1}{1 + e^{-B(\tau-\tau_0)}}; \quad 1 - f = \frac{1}{1 + e^{B(\tau-\tau_0)}}. \tag{19}$$

A general analytical expression for the n-th-order derivative of the function $T_0(\tau)$ is successfully obtained comparatively simply for such a choice of $f(\tau)$. The final form of the approximate dependence for $dT_0/d\tau$ is the following:

$$\frac{dT_0}{d\tau} = -D_1\frac{e^{-B_1\tau}}{1 + e^{B(\tau-\tau_0)}} - D_2\frac{e^{-B_2\tau}}{1 + e^{-B(\tau-\tau_0)}}. \tag{20}$$

The function (20) is shown in Fig. 3. The dependence $T_0(\tau)$ is obtained by integrating (20):

$$T_0(\tau) = T_i - D_1\int_0^\tau \frac{e^{-B_1\tau}\,d\tau}{1 + e^{B(\tau-\tau_0)}} - D_2\int_0^\tau \frac{e^{-B_2\tau}\,d\tau}{1 + e^{-B(\tau-\tau_0)}}. \tag{21}$$

The computations which were carried out showed that for real experimental curves $T_0(\tau)$ the error in the approximation (20) does not exceed the error in determining the derivative $dT_0/d\tau$.

A general analytical expression for the n-th derivative of the function $T_0(\tau)$ was obtained to derive the computational formulas on the basis of the dependences (2) and (3):

$$\frac{d^nT_0}{d\tau^n} = \frac{d^{n-1}}{d\tau^{n-1}}\left(\frac{dT_0}{d\tau}\right) = \sum_{k=0}^{n-1} C_{n-1}^k\,\{B^k\varphi_k(U_1)(-B_1)^{n-1-k}V_1 + (-B)^k\varphi_k(U_2)(-B_2)^{n-1-k}V_2\}, \tag{22}$$

where

$$U_1 = \frac{1}{1 + e^{B(\tau-\tau_0)}}; \quad U_2 = \frac{1}{1 + e^{-B(\tau-\tau_0)}};$$

$$V_1 = -D_1e^{-B_1\tau}; \quad V_2 = -D_2e^{-B_2\tau}; \quad \varphi_0(U) = U;$$

$$\varphi_k(U) = (1 - U)\sum_{s=1}^{k} a_{ks}U^s \quad (k \geqslant 1); \qquad a_{ks} = s\sum_{m=1}^{s}(-1)^{m-1}C_{s-1}^{m-1}m^{k-1}.$$

The coefficients a_{ks} can also be calculated by means of the recursion formula

$$a_{k+1,s} = s(a_{ks} - a_{k,s-1}) \quad (1 < s < k + 1);$$

$$a_{k+1,1} = 1; \quad a_{k+1,k+1} = (-1)^{k+1}(k + 1)!$$

Estimates show that $B_1 \ll B_2 \ll B$ always. Hence, (22) must be considered as a power series in the parameter B, i.e.,

$$\frac{d^n T_0}{d\tau^n} = \sum_{k=0}^{n-1} h_{nk} B^k. \tag{23}$$

Substituting (23) into (2) and (3) and changing the order of summation of the series, we obtain

$$T_0 - T_\mathrm{w} = -\sum_{n=1}^{\infty} E_n \sum_{k=0}^{n-1} h_{nk} B^k = \sum_{k=0}^{\infty} G_k B^k, \tag{24}$$

$$q = -\sum_{n=1}^{\infty} F_n \sum_{k=0}^{n-1} h_{nk} B^k = \sum_{k=0}^{\infty} H_k B^k, \tag{25}$$

where

$$G_k = -\sum_{n=k+1}^{\infty} h_{nk} E_n; \; H_k = -\sum_{n=k+1}^{\infty} h_{nk} F_n. \tag{26}$$

Formulas (24) and (25) are infinite power series of the approximation parameters for which the sums and residuals can be determined by the usual methods of the theory of numerical series.

Therefore, to process test data in an investigation of transition boiling under nonstationary cooling conditions, it is recommended to obtain the experimental time dependence of $dT_0/d\tau$, to approximate it by (20), and to determine the heat flux density q and the heat-exchange surface temperature by means of (24) and (25).

NOTATION

A	is the coefficient in (5);
B	is the coefficient in (19);
Bi	is the Biot criterion;
c	is the wall specific heat;
D	is the coefficient in (14);
E	is the coefficient in (2);
F	is the coefficient in (3);
f	is the fraction of surface occupied by liquid;
q	is the specific heat flux;
T	is the temperature;
α	is the heat transfer coefficient;
δ	is the wall thickness;
ρ	is the density;
τ	is the time.

Subscripts

l	is the liquid;
s	is the saturation line;
i	is the initial;
fl	is the film;
nu	is the nucleate;
w	is the wall.

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MOLECULAR THERMODYNAMIC METHOD OF CORRELATING HEAT TRANSFER IN LIQUID BOILING

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A correlation is obtained for heat transfer in boiling of various liquids. A classification of materials in similarity groups is proposed.

The present thermodynamic method for analyzing the vapor-generation process has been used successfully to develop important aspects of theory and to calculate many thermal processes. Boiling involves very complex physical processes which depend on thermodynamic, hydrodynamic, and molecular factors.

The well-established correlations of Kruzhilin, Kutateladze, Borishanskii, Tolubinskii, Labunstov, et al., based on the thermodynamic method of analyzing the process, do not allow correlation of test data on heat transfer in bubble boiling of liquids with sharply differing physical properties and process conditions.

The method proposed by I. I. Novikov and V. M. Borishanskii for obtaining correlations, based on using the thermodynamic law for the respective states, although it enlarges the capability to calculate the influence of physical properties of the boiling medium, does not however provide a broad correlation of heat transfer in boiling of liquids under different hydrodynamic process conditions.

The reasons for the unsatisfactory correlation with methods of thermodynamic analysis of test data on heat transfer in bubble boiling, in our opinion, lie in the deviation of individual properties of substances from the general thermodynamic law for the respective states [8, 10, 52, 53], in the incomplete allowance for hydrodynamic process conditions and the effect of the molecular properties of the material, particularly the intermolecular interactions in phase transition, and also in the fact that existing methods for allocating substances into thermodynamically similar groups do not take into account certain properties which characterize the behavior of different substances in the vapor-generation process.

It is evident that the thermodynamic method of analysis, while remaining important, has lost its exclusive feature and must be supplemented. The supplement may take the form of analysis of the boiling process allowing for the molecular characteristics of the system.

The main content of the molecular thermodynamic method is the fact that the boiling process is considered from the viewpoint of simultaneous interaction of macro- and microparticles of material; the allocation of boiling substances to similarity groups is carried out on the basis of the influence of the thermodynamic properties of the substance in the corresponding process states, and also the nature of interaction of molecules

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